ГЕОЛОГІЧНА ІНФОРМАТИКА

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THE ADVANCED ALGORITHM OF STATISTICAL SIMULATION OF SEISMIC NOISE IN THE MULTIDIMENSIONAL AREA FOR DETERMINATION THE FREQUENCY CHARACTERISTICS OF GEOLOGICAL ENVIRONMENT

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The article is devoted to the theory and methods of random process and field statistical simulation on the basis of their spectral decomposition and modified Kotelnikov-Shennon interpolation sums, as well as using these methods in environmental geophysical monitoring. The problem of statistical simulation of the multivariate random fields (homogeneous in time and homogeneous isotropic with respect to the n other variables) is considered for introducing into seismological researches for determination the frequency characteristics of geological environment. Statistical model and advanced numerical algorithm of simulation realizations of such random fields are built on the basis of modified interpolation Kotelnikov-Shennon decompositions for generating the adequate realizations of seismic noise. Real-valued random fields ξ (t, x), t ∈ R, x ∈ R ⁿ, that are homogeneous with respect to time and homogeneous isotropic with respect to spatial variables in the multidimensional spase are studied. The problem of approximation of such random fields by random fields with a bounded spectrum is considered. An analogue of the Kotelnikov–Shannon theorem for random fields with a bounded spectrum is presented. Improved estimates of the mean-square approximation of random fields in the space R × R n by a model constructed with the help of the spectral decomposition and interpolation Kotelnikov–Shannon formula are obtained. Some procedures for the statistical simulation of realizations of Gaussian random fields with a bounded spectrum that are homogeneous with respect to time and homogeneous isotropic with respect to spatial variables in the multidimensional spase are developed. There has been proved theorems on the mean-square approximation of homogeneous in time and homogeneous isotropic with respect to the n other variables random fields by special partial sums. A simulation method was used to formulate an advanced algorithm of numerical simulation by means of this theorems. The spectral analysis methods of generated seismic noise realizations are considered. There have been developed universal methods of statistical simulation (Monte Carlo methods) of multi parameters seismology data for generating of seismic noise on 2D and 3D grids of required detail and regularity.

Keywords: statistical simulation, spectral analyzes, seismic noise.

Introduction. This article describes the problem of improved statistical simulation algorithm for random field realizations with a limited spectrum which depends on time and was set in the multidimensional variables area for implementation into seismological research to determine the frequency characteristics of geological environment under the building sites. The model was built and based on improved estimates of random field mean approximation errors the improved algorithm was formulated by this model for numerical simulation of field realizations that are adequate to realizations of seismogram's noises.

It is a further theoretical generalization solved in papers [5-11] for problems concerning the increase of variables space dimensionality, where random field domain with the limited spectrum is focused. This generalization direction development is important because of necessarily to use the proposed method for statistical modeling of random fields with a limited spectrum that depend on the time and are set in the multidimensional variables area, where was added dimensionality value of one or more influential parameters additionally to the spatial coordinates.

Practically it is important to use the statistical simulation realizations of such random fields for the release of seismic noise dependent on one or more significant parameters and external influence, and to obtain appropriate estimations for the frequency characteristics of three-dimensional geological environment observation area. These estimations should be considered in the construction of different objects to ensure the building's solidity.

As can be seen from the articles ([14, 16-18, 20] and others), models and algorithms for numerical simulation of random processes and fields based on Fourier transform, Fourier-Bessel and series of sinc function (interpolation Kotelnikov-Shannon formula) are relatively recently applying in geological sciences.

The article describes the application prospects of constructed models and algorithms for statistical modeling of random fields based on a decomposition into modified Kotelnikov-Shannon interpolation series for seismic noise research problem, which depend on one or more critical parameters for the purpose of determining the frequency characteristics of the geological environment under the building sites in a single, two- or three-dimensional observation area.

1. The spectral decompositions and modified interpolation Kotelnikov-Shennon series

It is recommended to use the approach is developed on the basis of spectral decomposition of random fields, see [15], and modified Kotelnikov-Shennon theorem for random fields with a bounded spectrum which are homogeneous in time and homogeneous isotropic with respect to the other coordinates for the statistical simulation of observed seismogram's noises which depend on one or several important parameters.

Consider the following results that are proved on the basis of mentioned theory.

1.1 Time homogeneous and homogeneous isotropic with respect to the spatial variables random fields

Consider a real valued mean square continuous random field $\xi(t,x), t\in R, x\in R^n$, in $R\times R^n$ which is time homogeneous and homogeneous isotropic with respect to the other variables. This means that

1) $E\xi(t,x)=const$ for all $t\in R$ and $x\in R^n$ (we assume that $E\xi(t,x)=0$),

2) $E\xi(t,x)\xi(s,y)=B(t-s,p)$ for all $t,s\in R$ and for all $x,y\in R^n$, where $B(\tau,\rho)$ is a correlation function that

depends on the shift of the time $\tau = t - s$ and distance between the vectors and , that is on ρ.

The correlation function of a real valued random field $\xi(t,x)$ in $R \times R^n$ which is homogeneous with respect to time t and homogeneous isotropic with respect to the spatial variables admits the following integral representation, see [15, p. 11], as

$$B(t-s,p) = \int_{-\infty}^{+\infty}\int_0^{+\infty} e^{1(t-s)u} Y_n(\lambda\rho)\Phi(du,d\lambda), \quad (1)$$

where $Y_n(z) = 2^{\frac{n-2}{2}}\Gamma\left(\frac{n}{2}\right)J_{\frac{n-2}{2}}(z)z^{-\frac{n-2}{2}}$ and $\Phi(du,d\lambda)$ is a spatial-temporal spectral measure on Borel sets $(-\infty,+\infty)\times(0,+\infty)$, $J_n(x)$ is the Bessel function of the first kind of order.

The next statement for the spectral decomposition of such random field in $R \times R^n$ is mentioned in [15, p. 11].

Theorem 1. *A mean square continuous random field $\xi(t,x)$ in $R \times R^n$ which is time homogeneous and homogeneous isotropic with respect to the other variables admits the following spectral decomposition*

$$\xi(t,\rho,\theta_1,...,\theta_{n-2},\varphi) = c_n \sum_{m=0}^{\infty}\sum_{l=1}^{h(m,n)} S_m^l(\theta_1,...,\theta_{n-2},\varphi)\int_{-\infty}^{+\infty} e^{itu}\int_0^{+\infty}\frac{J_{\frac{n-2}{2}+m}(\lambda\rho)}{(\lambda\rho)^{\frac{n-2}{2}}} Z_m^l(du,d\lambda), \quad (2)$$

where $(\rho,\theta_1,...,\theta_{n-2},\varphi)$ are spherical coordinates of point $x, S_m^l(\theta_1,...,\theta_{n-2},\varphi)$ are orthonormal spherical harmonics of order m, number $h(m,n) = \frac{(2m+n-2)(m+n-3)!}{(n-2)!m!}$ is the number of linearly independent spherical harmonics of order n, $c_n^2 = 2^{n-1}\Gamma\left(\frac{n}{2}\right)\pi^{\frac{n}{2}}$ is a constant and $\{Z_m^l(\bullet)\}$ are sequences of real valued orthogonal random measures on Borel subsets of the set $(-\infty,+\infty)\times[0,+\infty)$ such that

$$EZ_m^l(B_1) = 0, EZ_m^l(B_1)Z_p^q(B_2) = \delta_m^p\delta_l^q\Phi(B_1 \cap B_2) \quad (3)$$

for all Borel subsets B_1 and B_2 of $R \times R_+$, $m,p = 0,1,...$ and $l,q = 1,2,...h(m,n)$; here *$\Phi(u,\lambda)$ is the spectral function of the random field.*

Moreover, the spectral measures $Z_m^l(B), m = 0,1,...,$ $l = 1,2,...h(m,n)$, are uniquely determined with probability one by the following relations

$$Z_m^l([\lambda_1,\lambda_2)\times[\gamma_1,\gamma_2)) =$$
$$= l.i.m.\int_{-T}^{T}\int_0^{+\infty}\int_{S_n}\left[\frac{e^{-l\lambda_2 t} - e^{-l\lambda_1 t}}{-it[\varphi_{m,\gamma_2}(\rho)]} - \varphi_{m,\gamma_1}(\rho)\right]\times \quad (4)$$
$$\times S_m^l(\theta_1,...,\theta_{n-2},\varphi)\xi(t,\rho,\theta_1,...,\theta_{n-2},\varphi)dm_n d\rho dt,$$

where $m_n(\bullet)$ is the Lebesgue measure in a unit sphere S_n of R $S_m^l(\bullet)$ are orthonormal spherical harmonics of order, and

$$\varphi_{0,\gamma}(\rho) = \frac{1}{c_n}\gamma^{\frac{n}{2}}\rho^{\frac{n-2}{2}}J_{\frac{n}{2}}(\gamma\rho),$$

$$\varphi_{m,\gamma}(\rho) = \frac{1}{c_n\rho}\left[(m+n-2)\rho\gamma J_{m+\frac{n-2}{2}}(\gamma\rho) + S_{\frac{n-2}{2},m+\frac{n}{2}-\rho}(\gamma\rho) - \gamma\rho J_{m+\frac{n}{2}-2}(\gamma\rho)S_{\frac{n}{2},m+\frac{n-2}{2}}(\gamma\rho) + 2^{\frac{n}{2}}\frac{\Gamma\left(\frac{m+n}{2}\right)}{\Gamma\left(\frac{m}{2}\right)}\right]$$

for $m > 0$, $S_{\mu,\upsilon}(z)$ is the Lommel function.Let $c_m^v(z)$ are Gegenbauer polynomials, see [2, p. 177], which are defined in terms of their generating function

$$(1-2zt+t^2)^{-v}\sum_{m=0}^{\infty} c_m^v(z)t^m, |t| < \left|z+\sqrt{z^2-1}\right|.$$

The correlation function of a mean square continuous random field $\xi(t,x)$ in $R \times R^n$ which is homogeneous with respect to time t and homogeneous isotropic with respect to the other variables admits the following expansion, see [11],

$$B(t-s,\rho) = B(t-s,|x_1-x_2|) = c_n^2\int_0^{+\infty}\int_{-\infty}^{+\infty}\sum_{m=0}^{\infty}\sum_{l=1}^{h(m,n)} S_m^l(\theta_1',...,\theta_{n-2}',\varphi')\times$$
$$\times S_m^l(\theta_1'',...,\theta_{n-2}'',\varphi'')\frac{J_{\frac{n-2}{2}+m}(\gamma\rho_1)}{(\gamma\rho_1)^{\frac{n-2}{2}}}\frac{J_{\frac{n-2}{2}+m}(\gamma\rho_2)}{(\gamma\rho_2)^{\frac{n-2}{2}}}e^{l(t-s)u}\Phi(du,d\lambda), \quad (5)$$

where $x_1 = \theta_1,...,\theta_{n-2},\ \varphi,\ x_2 = \theta_1,...,\theta_{n-2},\ \varphi$.

If one considers the "restriction" of the random field $\xi(t,x)$ to the sphere of a fixed radius $r = \rho$ then the correlation function of such random process can be written as

$$E\xi(t,r,\theta_1',...,\theta_{n-2}',\varphi')\xi(s,r,\theta_1'',...,\theta_{n-2}'',\varphi'') = c_n^2\sum_{m=0}^{\infty}\sum_{l=1}^{h(m,n)} S_m^l(\theta_1',...,\theta_{n-2}',\varphi)\times$$
$$\times S_m^l(\theta_1'',...,\theta_{n-2}'',\varphi'')\int_{-\infty}^{+\infty}\int_0^{+\infty} e^{l(t-s)u}\frac{J_{\frac{n-2}{2}+m}^2(\gamma r)}{(\gamma r)^{n-2}}\Phi(du,d\lambda). \quad (6)$$

Than follows that spectral coefficients are expressed in terms of the spectral function as

$$b_m(t-s,r)=\int_{-\infty}^{+\infty}\int_0^{+\infty} e^{l(t-s)u}\frac{J^2_{\frac{n-2}{2}+m}(\lambda r)}{(\lambda r)^{n-2}}\Phi(du,d\lambda) \quad (7)$$

Consider the following decomposition of a mean square continuous random field $\xi(t,x)$ which is homogeneous with respect to time and homogeneous isotropic with respect to the other variables, that is

$$\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)= =\sum_{m=0}^{\infty}\sum_{l=1}^{h(m,n)} S_m^l(\theta_1,...,\theta_{n-2},\varphi)\xi_m^l(t,r), \quad (8)$$

where

$$\xi_m^l(t,r)=\int_{-\infty}^{+\infty}\int_0^{+\infty} e^{l(t-s)u}\frac{J_{\frac{n-2}{2}+m}(\lambda r)}{(\lambda r)^{\frac{n-2}{2}}}Z_m^l(du,d\lambda),$$

$m=0,1,...;\ l=0,1,...h(m,n).$

Note that we use in (8) a notation similar to (2).

Since $E\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)=0$ we have $E\xi_m^l(t,r)=0.$

Theorem 2. If $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ is a random field in $R\times R^n$ which is homogeneous in time and homogeneous isotropic with respect to the spatial variables $r,\theta_1,...,\theta_{n-2},\varphi$ then

$$E\xi_m^l(t,r)\xi_q^k(s,r)=\delta_m^q\delta_l^k b_m(t-s,r), \quad (9)$$

where δ_l^k is the Kronecker symbol, $\{b_m(t-s,r)\}$ is a sequence of positive definite kernels in $R\times R_+$ of the form (7) and such that $\sum_{m=0}^{\infty}h(m,n)b_m(0,r)<\infty.$

The variance of the random field $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ is expressed in terms of the spectral coefficients as

$$D\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)= =2^{n-2}\Gamma^2\left(\frac{n}{2}\right)\sum_{m=0}^{\infty}h(m,n)b_m(0,r).\blacksquare \quad (10)$$

Consider particular cases which have been studied in [6, 8, 10]. Let and $h(0,2)=1,\ h(m,2)=2$ than the variance of the random field $\xi(t,r,\varphi)$ is expressed in terms of the spectral coefficients as

$$D\xi(t,r,\varphi)=b_0(0,r)+2\sum_{m=0}^{\infty}b_m(0,r),$$

where $b_0(0,r)=\int_{-\infty}^{+\infty}\int_0^{+\infty}J_m(\lambda r)\Phi(du,d\lambda).$

Let and $h(m,3)=2m+1$ than the variance of the random field $\xi(t,r,\theta,\varphi)$ is expressed in terms of the spectral coefficients as

$$D\xi(t,r,\theta,\varphi)=\pi\sum_{m=0}^{\infty}\left(m+\frac{1}{2}\right)b_m(0,\rho),$$

where $b_0(0,r)=\int_{-\infty}^{+\infty}\int_0^{+\infty}\frac{J^2_{\frac{1}{2}+m}(\lambda r)}{\lambda r}\Phi(du,d\lambda).$

Thus the expansion (8) can be used for statistical simulation of random fields in $R\times R^n$ which are homogeneous with respect to time and homogeneous isotropic with respect to the variables $r,\theta_1,...,\theta_{n-2},\varphi$ if the spectral function (or correlation function) is specified.

1.2 Time homogeneous random fields with a bounded spectrum

Consider a random field $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ in $R\times R^n$. We say that $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ is a random field with a bounded spectrum if all its spectral measures $Z_m^l(B)$ in (4) are concentrated in $[-\tilde\omega,\tilde\omega]\times R_+,\tilde\omega>0.$

Let $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi),\ t\in R,\ r\in R_+,\ \theta_1\in[0,\pi],\ i=1,...,n-2,\ \varphi\in[0,2\pi]$ be a random field in $R\times R^n$ which is time homogeneous and homogeneous isotropic with respect to the variables $r,\theta_1,...,\theta_{n-2},\varphi$. Assume that the spectrum $\Phi(U,\Lambda)$ of the field ξ is bounded with respect to time t, $U\subset[-\tilde\omega,\tilde\omega],\ \Lambda\subset R_+;$ and let $\Phi(U,\Lambda)$ be concentrated in $[-\tilde\omega,\tilde\omega]\times R_+$.

Let ω be an arbitrary number such that $\omega>\tilde\omega$. Put

$$\xi_N(t,r,\theta_1,...,\theta_{n-2},\varphi)= =\sum_{k=-N}^{N}\xi\left(\frac{k\pi}{\omega},r,\theta_1,...,\theta_{n-2},\varphi\right)\frac{\sin\omega\left(t-\frac{k\pi}{\omega}\right)}{\omega\left(t-\frac{k\pi}{\omega}\right)}. \quad (11)$$

Then the following assertion holds, see [11].

Theorem 3. *Let $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ be a random field in $R\times R^n$ which is time homogeneous and homogeneous isotropic with respect to the variables $r,\theta_1,...,\theta_{n-2},\varphi$. If the spectrum of $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ is bounded in time t then the mean square approximation with the help of partial sum (11) is such that*

$$E\left|\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)-\xi_N(t,r,\theta_1,...,\theta_{n-2},\varphi)\right|^2\le \le\frac{\gamma^2(t)}{N^2\left(1-\frac{\tilde\omega}{\omega}\right)^2}2^{n-2}\Gamma^2\sum_{m=0}^{\infty}h(m,n)\bar b_m(0,r). \quad (12)$$

where

$$\bar b_m(0,r)=\int_{|u|\le\tilde\omega}\int_0^{+\infty}\frac{J^2_{\frac{n-2}{2}+m}(\lambda r)}{(\lambda r)^{n-2}}\Phi(du,d\lambda). \quad (13)$$

Corollary. *Let $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ be a random field in $R\times R^n$ whose spectrum is bounded in time t. Then $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ admits the following Kotelnikov–Shannon decomposition:*

$$\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)= =\sum_{k=-\infty}^{\infty}\xi\left(\frac{k\pi}{\omega},r,\theta_1,...,\theta_{n-2},\varphi\right)\frac{\sin\omega\left(t-\frac{k\pi}{\omega}\right)}{\omega\left(t-\frac{k\pi}{\omega}\right)}, \quad (14)$$

where the series on the right hand side of (14) converges in the mean square sense for $\omega>\tilde\omega$.

2. The improved mean square estimate for the approximation and advanced procedure for the statistical simulation

The Kotelnikov–Shannon decomposition (14) of random fields in $R\times R^n$ with a bounded spectrum which are time homogeneous and homogeneous isotropic with respect to the other variables it is possible to use for the statistical simulation of such random fields with their defined statistical characteristics. By the simulating is important to improve the estimate of the mean square approximation (12) for using it in the advanced procedure for the numerical simulation realizations of these random fields. The variants of such estimates are obtained in the next theorems.

We use partial sum (8) and partial sum of decomposition (14) for a random field $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ which are time homogeneous and homogeneous isotropic with respect to the variables $r,\theta_1,...,\theta_{n-2},\varphi$ to construct a model for such field if its spectrum is bounded with respect to time t and concentrated on an interval $[-\tilde{\omega},\tilde{\omega}]\times R_+$.

The following partial sum is taken as an approximation model of such a random field

$$\xi_{N,M}(t,r,\theta_1,...,\theta_{n-2},\varphi)=c_n\sum_{k=-N}^{N}\frac{\sin\omega\left(t-\frac{k\pi}{\omega}\right)}{\omega\left(t-\frac{k\pi}{\omega}\right)}\sum_{m=0}^{M}\sum_{l=1}^{h(m,n)}S_m^l(\theta_1,...,\theta_{n-2},\varphi)\xi_m^l\left(\frac{k\pi}{\omega},r\right), \tag{15}$$

where $\xi_m^l\left(\frac{k\pi}{\omega},r\right), m,p=0,1,...M;\quad k,q=\overline{-N,N};$ $l,s=1,...h(m,n)$, is a sequence of Gaussian stochastic processes such that

$$E\xi_m^l\left(\frac{k\pi}{\omega},r\right)=0,\ E\xi_m^l\left(\frac{k\pi}{\omega},r\right)\xi_p^s\left(\frac{q\pi}{\omega},r\right)= \\ =\delta_l^s\delta_p^m\tilde{b}_m\left(\frac{(k-q)\pi}{\omega},r\right)...\ ...\ ...\ ...\ ... . \tag{16}$$

It is known that $\{b_m(t-s,r)\}$ is a sequence of positive definite kernels in $R\times R_+$ that can be calculated by means of the spatial-temporal spectrum $\Phi(du,d\lambda)$ of the random field $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ by expression (13) and such that satisfies following condition $\sum_{k=-\infty}^{\infty}h(m,n)b_m(0,r)<\infty.$

For formulating the advanced procedure of numerical simulation the realizations of Gaussian random field $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ which is time homogeneous and homogeneous isotropic with respect to the variables $r,\theta_1,...,\theta_{n-2},\varphi$ whose spectrum is bounded in it is necessary to derive more improved mean square estimate for the approximation of such random field by its approximation model (15). Such results are deduced in the next theorems.

Theorem 4. *The mean square estimate for the approximation of random field* $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)=\xi(t,x)$ *in* $R\times R^n, n\ge 3$ *which is time homogeneous and homogeneous isotropic with respect to the variables* $r,\theta_1,...,\theta_{n-2},\varphi$ *whose spectrum is bounded in t by its approximation model (15) assumes following expression*

$$E\left|\xi(t,x)-\overline{\xi}_{N,M}(t,x)\right|^2\le\frac{\gamma^2(t)}{N^2}\frac{2\tilde{\mu}_0}{\left(1-\frac{\tilde{\omega}}{\omega}\right)^2}+\frac{2r^{2M+2}}{((M+1)!)^2}\frac{\Gamma\left(\frac{n}{2}\right)\Gamma\left(\frac{2M+n+1}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)\Gamma\left(\frac{2M+n+2}{2}\right)}\tilde{\mu}_{2M+2}<\varepsilon, \tag{17}$$

where

$$\gamma(t)=\frac{4\left(\frac{\omega}{\pi}|t|+1\right)}{\pi} \tag{18}$$

$$\tilde{\mu}_k=\int_{-\tilde{\omega}}^{+\tilde{\omega}}\int_0^{+\infty}\lambda^k\Phi(du,d\lambda). \tag{19}$$

Using the results from [10] another improved mean square estimate for the approximation of random field $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ by the model (15) is obtained. Following theorems 5 and 6 are proved.

Theorem 5. *The mean square estimate for the approximation of random field* $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)=\xi(t,x)$ *in* $R\times R^n, n\ge 3$ *which is time homogeneous and homogeneous isotropic with respect to the variables* $r,\theta_1,...,\theta_{n-2},\varphi$ *whose spectrum is bounded in t by its approximation model (15) is written as follows*

$$E\left|\xi(t,x)-\overline{\xi}_{N,M}(t,x)\right|^2\le 2\overline{\mu}_0\frac{L_0^2(t)\omega^2}{(\omega-\tau)^2N^2}+\frac{2r^{2M+2}}{((M+1)!)^2}\frac{\Gamma\left(\frac{n}{2}\right)\Gamma\left(\frac{2M+n+1}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)\Gamma\left(\frac{2M+n+2}{2}\right)}\tilde{\mu}_{2M+2}<\varepsilon, \tag{20}$$

where $\omega>v=\sup_{u\in\Lambda}|u|$ is an arbitrary number, Λ *is an interval* $[-\tilde{\omega},\tilde{\omega}]$,

$$L_0(t)=\frac{2}{1-e^{-\pi}}\left(\frac{2}{\pi}\right)|\sin\omega t|. \tag{21}$$

Theorem 6. *The mean square estimate for the approximation of random field* $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)=\xi(t,x)$ *in* $R\times R^n, n\ge 3$ *which is time homogeneous and homogeneous isotropic with respect to the variables* $r,\theta_1,...,\theta_{n-2},\varphi$ *whose spectrum is bounded in t by its approximation model (15) admits following expression*

$$E\left|\xi(t,x)-\overline{\xi}_{N,M}(t,x)\right|^2 \le 2\overline{\mu}_0 \frac{8}{(2N-1)\pi^2}+\frac{2r^{2M+2}}{((M+1)!)^2}\frac{\Gamma\left(\frac{n}{2}\right)\Gamma\left(\frac{2M+n+1}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)\Gamma\left(\frac{2M+n+2}{2}\right)}\tilde{\mu}_{2M+2}<\varepsilon. \qquad (22)$$

The improved mean square estimate for the approximation of a random field $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ in $R\times R^n, n\ge 3$ which is time homogeneous and homogeneous isotropic with respect to the variables $r,\theta_1,...,\theta_{n-2},\varphi$ whose spectrum is bounded in t by a model (15) are derived. Thus the lemma 8 [3, p. 117] is used.

Applying previous principles of expansion and thinking as well as to the estimates (17), (20), (22), the similar three mean square estimates for the approximation of a random field $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ in $R\times R^n, n=2$ which is time homogeneous and homogeneous isotropic with respect to the variables $r,\theta_1,...,\theta_{n-2},\varphi$ whose spectrum is bounded in t by a model (15) are obtained as

$$E\left|\xi(t,x)-\tilde{\xi}_{N,M}(t,x)\right|^2 \le \frac{\gamma^2(t)}{N^2}\frac{2\tilde{\mu}_0}{\left(1-\frac{\tilde{\omega}}{\omega}\right)^2}+\frac{2}{\pi M}\left(\frac{1}{2}r\tilde{\mu}_1+r^2\tilde{\mu}_2\right)<\varepsilon, \qquad (23)$$

$$\left|\xi(t,x)-\tilde{\xi}_{N,M}(t,x)\right|^2 \le 2\tilde{\mu}_0\frac{L_0^2(t)\omega^2}{(\omega-\tau)^2N^2}+\frac{2}{\pi M}\left(\frac{1}{2}r\tilde{\mu}_1+r^2\tilde{\mu}_2\right)<\varepsilon, \qquad (24)$$

$$E\left|\xi(t,x)-\tilde{\xi}_{N,M}(t,x)\right|^2 \le 2\tilde{\mu}_0\frac{8}{\pi^2(2N-1)}+\frac{2}{\pi M}\left(\frac{1}{2}r\tilde{\mu}_1+r^2\tilde{\mu}_2\right)<\varepsilon. \qquad (25)$$

Where r is a polar radius ω is an arbitrary number such that $\omega > v=\sup_{u\in\Lambda}|u|$.

Then the procedure for the statistical simulation the realizations of a Gaussian random field which is time homogeneous and homogeneous isotropic with respect to variables $(r,\theta_1,...,\theta_{n-2},\varphi)$ can be stated as follows if its spectrum is bounded in *t*.

The procedure:

1. According to a prescribed accuracy $\varepsilon>0$, choose positive integer numbers N and M for the model (15) by using one of the following inequalities (23), (24), (25) (in case $n=2$) and (17), (20), (22) (in case).

2. For a fixed polar radius r, generate values of the Gaussian stochastic processes $\xi_m^l\left(\frac{k\pi}{\omega},r\right), m=0,1,...M;$ $k=\overline{-N,N};\ l=1,...,h(m,n),$ such that satisfy conditions (16).

3. Evaluate the expression in (15) at a given point , by substituting the numbers N and M and values of Gaussian stochastic processes evaluated in steps 1 and 2.

4. Check whether the realization of the stochastic random field $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ in the grid points in area of observation generated in step 3 fits the data of this random field by testing the corresponding statistical characteristics.

3. Practical use of field simulation with space-time correlation function

Different approaches [10] can be applied for practical use of the algorithm and model (18) for numerical simulation of real and homogeneous in time t implementations, that are homogeneous and isotropic with respect to variables $r,\theta_1,...,\theta_{n-2},\varphi$ on $R\times R^n$ of random fields $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$, which have a limited spectrum and space-time correlation function $B_z(\tau,\rho)$. It should be noted that models of space-time correlation structure are divided into two types: first takes into account the distribution of the spatial and time components and other with no such distribution. Work [7] gives an example of application and most commonly used models, namely metric model, linear model, model of space-time covariance product and model of product and sum.

Different approach can also be used to simulate space-time correlation that allows classifying the undivided space-time stationary covariance functions. This approach is based on the frequency representation of covariance function.

Practical use example in seismology of developed algorithm and numerical simulation model for real and homogeneous in time t implementations, two-dimensional homogeneous isotropic random fields with a limited spectrum and space-time correlation function $B_z(\tau,\rho)$ by method which divides the spatial and time components with product-sum formula described in [6].

The realization value arrays of random process $\xi(t,\rho,\varphi)$ (ρ,φ – fixed) were simulated as noise seismograms for each observation point on each component: EW, NS, and Z. They give important information about soil vibration properties within the territory of building and operating sites. These properties are also required for design of new antiseismic buildings and constructions, and providing earthquake resistance of existing buildings in order to avoid dangerous resonance effects. Random disturbances from random external factors were removed from the simulated noise seismograms by statistical averaging filters. These disturbances include vibrations caused by the movement of trains or heavy car and so on. The adequacy of value array results from the simulated by statistical methods noise seismograms were tested on real seismograms from observation points.

The statistical modeling method of random fields can also solve a major simulation problem of the artificial realization of noise seismograms that simulated for imaginary observation points, located between the real points of observation or at a small distance from them. All values except time t, in $\xi(t,r,\theta_1,...,\theta_{n-2},\varphi)$ are fixed and spectral analysis was performed of random process realizations. Amplitude and phase spectra of such noise realization may be used to obtain the frequency characteristics of the geological environment under building sites, describing its ability to change (increase or decrease) the amplitude of the seismic waves during earthquakes [1, 8]. Numerical simulation of soil strata frequency characteristics in some cases can significantly reduce the cost of seismic zoning of building sites by reducing the number of instrumental observation points for earthquakes, explosions and microseism.

4. Spectral analysis of generated noise

Frequency characteristic estimates for the geological environment with multidimensional observation area (under construction sites) can be obtained by calculating and constructing the amplitude and phase spectra of noise in seismogram observation points in that area, considering fixed all arguments except time [4]. Calculations of the amplitude and phase spectra can be made by direct method [1, p. 179], i.e. periodogram method. Then based on these results the spectral ratio of the Earth crust was build, which is independent of the spectrum of incident seismic waves, but determined entirely by the geological environment structure under the observation point.

Those spectral methods that use frequency as an independent parameter provide information about the structure and filtration properties of the upper crust layers, because any medium is a filter that due to resonance and reverberation effects increases the oscillation amplitude for some frequencies and reduces for the other [1, p. 270]. The ability to simulate the effects depends on amplitude and phase frequency characteristics of the geological environment for observation points situated under building sites and operating platforms, allows studying the geological section features and predicting places where significant increase in the seismic oscillation intensity is possible due to resonance effects and oscillation field interference nodes.

Among the many ways to eliminate the influence of various factors that affect the spectrum shape of seismic waves during earthquakes, explosions and microseism except that due to the influence of the upper crust section part, the way should be noted based on the use of the vertical $|S_Z(\omega)|$ component spectra relations to the horizontal $|S_N(\omega)|$ component. Spectra must be calculated for the same wave. This ratio is called the crust spectral ratio $T(\omega)$.

$$|S_Z(\omega)| / |S_N(\omega)| = T(\omega).$$

The ratio $T(\omega)$ is independent of the spectrum of incident seismic waves, but determined entirely by the geological environment structure under the observation point. Figures 1 a and 1b show graphs of amplitude spectra $|S(\omega)|$ for the initial simulated noise realization for imaginary observation point with the oscillation components Z and NS respectively, Figure 1 represents earth crust transmission ratio graph $T(\omega)$, that was built on the smoothed amplitude spectrum ratio of simulated noise seismogram realization on the fluctuation Z-component to the similar spectrum of fluctuation component NS for an observation point.

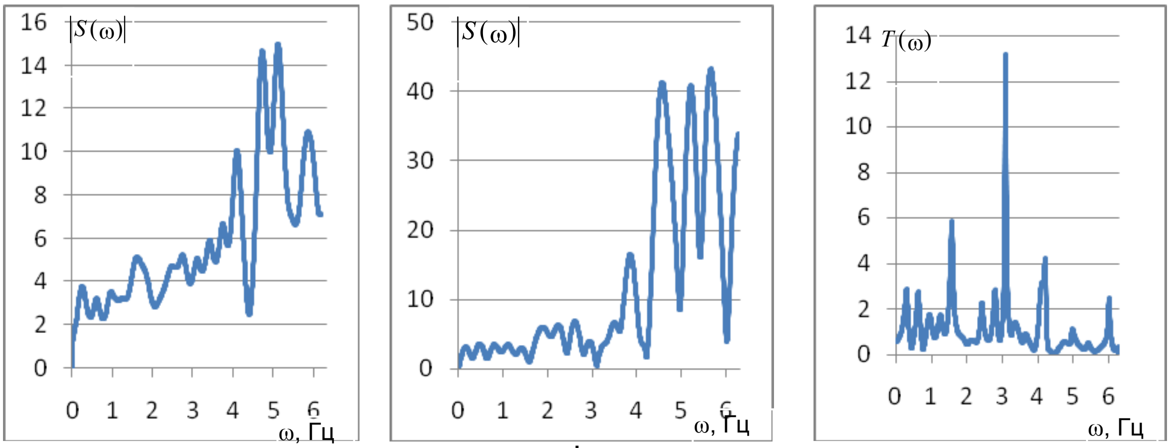

Fig. 1. Graphs of amplitude spectra $|S(\omega)|$ for simulated array noise realization for imaginary observation point on the component a) Z and b) NS; c) the graph of transmission ratio $T(\omega)$ for smoothed amplitude spectra of simulated noise realization for imaginary observation point

Interpretation of crust transmission ratio for these observations was conducted by comparing them with theoretical ratio calculated for well-known models of the upper section part. It should be noted that one of the important tools for impact assessment of the geological section upper part on seismic movements are widely known Nakamura method H/V or QTS (Quasi-Transfer Spectra), developed by Japanese scientist Yutaka Nakamura. The method uses records of microseismic noise registered for the horizontal and vertical oscillation components using borehole observations for the construction of a quasi-transmitting spectrum of soil strata. Nakamura method allows determining resonant eigen frequency of soil strata using spectrum ratios of horizontal and vertical components of the natural seismic noise. The maximum values of the microseism spectrum ratio of horizontal to vertical component are explained by multiple reflection of SH waves.

Figure 1 shows graph $T(\omega)$ of smoothed amplitude spectra transmission ratio for imaginary observation point that can be used to determine the increase of seismicity level on different parts of the building site, relative to the real observation point.

Conclusions. The model and algorithm of statistical simulation for time-homogeneous and multidimensionally homogeneous isotropic random fields with a limited spectrum were developed. These results continued research set in works [4-9] for modeling and generation method of noise seismogram implementations at flat observation area [5] and seismograms from three-dimensional observation area [6] and it is an important supplement to the Monte Carlo method used in geology and described in [14].

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ПОКРАЩЕНИЙ АЛГОРИТМ СТАТИСТИЧНОГО МОДЕЛЮВАННЯ СЕЙСМІЧНОГО ШУМУ В БАГАТОВИМІРНІЙ ОБЛАСТІ ЗМІННИХ ДЛЯ ВИЗНАЧЕННЯ ЧАСТОТНИХ ХАРАКТЕРИСТИК ГЕОЛОГІЧНОГО СЕРЕДОВИЩА

Робота присвячена подальшій розробці теорії та методів статистичного моделювання випадкових процесів та полів на основі їх спектральних розкладів та модифікованих інтерполяційних рядів Котельникова-Шеннона, а також застосуванню таких методів у задачах геофізичного моніторингу навколишнього середовища. Розглянуто задачу статистичного моделювання випадкових полів у багатовимірній області змінних (однорідних за часом та однорідних ізотропних за n іншими змінними) при впровадженні у сейсмологічні дослідження для визначення частотних характеристик геологічного середовища. Побудовано модель та сформульовано покращений алгоритм чисельного моделювання реалізацій таких випадкових полів на основі модифікованих інтерполяційних розкладів Котельникова-Шеннона для генерування адекватних реалізацій шуму сейсмограм. У статті вивчаються дійснозначні випадкові поля $\xi(t,x),\ \ t\in R,\ \ x\in R^n$ ***– однорідні за часом та однорідні ізотропні за просторовими змінними в багатовимірному просторі. Розглядається проблема апроксимації таких випадкових полів випадковими полями з обмеженим спектром. Для випадкових полів з обмеженим спектром встановлено аналог теореми Котельникова-Шеннона. Отримано вдосконалені оцінки середньоквадратичного наближення випадкових полів у просторі*** $R\times R^n$ ***моделлю, побудованою на основі спектрального розкладу та інтерполяційної формули Котельникова-Шеннона. Розроблено покращений алгоритм статистичного моделювання реалізацій гауссівських однорідних за часом та однорідних ізотропних за просторовими змінними в багатовимірному просторі випадкових полів з обмеженим спектром. Наведено теореми про оцінки середньоквадратичної апроксимації однорідних за часом та однорідних ізотропних за n іншими змінними випадкових полів частковими сумами рядів спеціального вигляду, за допомогою яких сформульовано покращений алгоритм чисельного моделювання реалізацій таких випадкових полів. Розглянуто способи проведення спектрального аналізу згенерованих реалізацій шуму сейсмограм. Розроблено універсальні методи статистичного моделювання (методи Монте-Карло) багатопараметричних сейсмологічних даних, які дають можливість вирішити проблеми генерування реалізацій шуму сейсмограм на площині та у тривимірному просторі на сітці необхідної детальності та регулярності.***

Ключові слова: статистичне моделювання, спектральний аналіз, сейсмічний шум.

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УЛУЧШЕННЫЙ АЛГОРИТМ СТАТИСТИЧЕСКОГО МОДЕЛИРОВАНИЯ СЕЙСМИЧЕСКОГО ШУМА В МНОГОМЕРНОЙ ОБЛАСТИ ПЕРЕМЕННЫХ ДЛЯ ОПРЕДЕЛЕНИЯ ЧАСТОТНЫХ ХАРАКТЕРИСТИК ГЕОЛОГИЧЕСКОЙ СРЕДЫ

Работа посвящена разработке теории и методологии статистического моделирования случайных процессов и полей на основе их спектральных разложений и модифицированных интерполяционных рядов Котельникова-Шеннона, а также применению таких методов в задачах геофизического мониторинга окружающей среды. Рассмотрена задача статистического моделирования случайных полей в многомерной области переменных (однородных по времени и однородных изотропных по n другими переменным) при внедрении в сейсмологические исследования для определения частотных характеристик геологической среды. Построена модель и сформулирован улучшенный алгоритм численного моделирования реализаций таких случайных полей на основании модифицированных интерполяционных разложений Котельникова-Шеннона для генерирования адекватных реализаций шума сейсмограмм. В статье изучаются действительнозначные случайные поля $\xi(t,x),\ t\in R,\ x\in R^n$ ***– однородные по времени и однородные изотропные по пространственным переменным в многомерном пространстве. Рассматривается проблема аппроксимации таких случайных полей случайными полями с ограниченным спектром. Для случайных полей полями с ограниченным спектром установлен аналог теоремы Котельникова-Шеннона. Получены усовершенствованные оценки среднеквадратического приближения случайных полей в пространстве*** $R\times R^n$ ***моделью, которая построена на основе спектрального разложения и интерполяционной формулы Котельникова-Шеннона. Разработан улучшенный алгоритм статистического моделирования реализаций гауссовских однородных по времени и однородных изотропных по пространственным переменным случайных полей с ограниченным спектром. Доказаны теоремы об оценке среднеквадратической аппроксимации однородных по времени и однородных изотропных по n другими переменным случайных полей частичными суммами рядов специального вида, при помощи которых сформулирован улучшенный алгоритм численного моделирования реализаций таких случайных полей. Рассмотрены способы проведения спектрального анализа сгенерированных реализаций шума сейсмограмм. Разработаны универсальные методы статистического моделирования (методы Монте-Карло) многопараметрических сейсмологичеких данных, которые дают возможность решить проблемы генерирования реализаций шума сейсмограмм на плоскости и в трехмерном пространстве на сетке необходимой детальности и регулярности.***

Ключевые слова: статистическое моделирование, спектральный анализ, сейсмический шум.